A Class of Cosmological Matrix Models

Miao Li
Institute of Theoretical Physics
Academia Sinica, P.O. Box 2735
Beijing 100080, P. R. China

and

Interdisciplinary Center of Theoretical Studies
Academia Sinica
mli@itp.ac.cn

We discuss a class of matrix models describing cosmology with a light-like singularity, generalizing the model proposed by Craps et al. in hep-th/0506180.

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Craps et al. recently proposed to study a simple cosmological background with a null singularity [1], this cosmology admits a matrix model description, thus lends itself to a rigorous study. An earlier example of cosmology with a null singularity is proposed in [2,3] and is later studied by many authors [4-6], they find that this singularity is highly unstable due to gravitational back-reaction. However, the model of Craps et al. seems to avoid this problem.

The construction of the model in [1] is simple. One starts in type IIA string theory with a flat string metric and a linear dilaton background $\phi = -Qx^+$, where x^+ is a light-like coordinate. The linear dilaton does not require modifying the critical dimension, since its linearity is along a null direction. Although the string metric is flat, the Einstein metric is nontrivial:

$$ds^2 = e^{Qx^+/2}(-2dx^+dx^- + (dx^i)^2). \tag{1}$$

For a positive Q, the metric contracts to a singularity at $x^+ = -\infty$, this is actually a curvature singularity. The corresponding 11 dimensional M theory metric also has a singularity at $x^+ = \infty$. The authors of [1] show that the singularity at $x^+ = -\infty$ lies in a finite geodesic distance, while the other singularity is at infinite distance. This background preserves half of total 32 supersymmetries, one expects that there is a control over the null singularity.

Since the string metric is flat, this time-dependent background admits a matrix string description, in which the Yang-Mills coupling constant is time-dependent. In fact, the authors of [1] show that this Yang-Mills theory can be regarded as one with a constant coupling on a world sheet with a time-dependent metric.

It goes without saying that this is an important observation, and it may provide the first example in which we can study a time-dependent background in a controlled fashion. Thus, it is interesting to ask whether this model is unique, or it has many cousins. In this note we will see that indeed there is a large class of such models.

We shall limit ourselves in M theory in this note. Consider metric

$$ds^2 = e^{2\alpha x^+}(-2dx^+dx^- + (dx^i)^2) + e^{2\beta x^+}(dx^a)^2, \tag{2}$$

where $i = 2, \ldots 10 - d$, $a = 11 - d, \ldots 10$, namely, there are d coordinates x^a, the total dimensions of spacetime is 11. The metric of [1] is given by taking $d = 1$, $\alpha = Q/3$ and $\beta = -2Q/3$. X^{10} is taken as the M theory circle, thus the metric reduces to the IIA flat string metric with a linear dilaton background.

To find a solution, we use an orthonormal basis

$$e^{\pm} = e^{\alpha x^+} dx^{\pm}, \quad e^i = e^{\alpha x^+} dx^1, \quad e^a = e^{\beta x^+} dx^a. \tag{3}$$

The non-vanishing components of spin connection are

$$\omega_{+-} = \alpha e^{-\alpha x^+} e^+, \quad \omega_{i+} = \alpha e^{-\alpha x^+} e^i, \quad \omega_{a+} = \beta e^{-\alpha x^+} e^a. \tag{4}$$

The non-vanishing curvature 2-forms are

$$\begin{aligned} R_{i+} &= \alpha^2 e^{-2\alpha x^+} e^i \wedge e^+, \\ R_{a+} &= \beta(2\alpha - \beta) e^{-2\alpha x^+} e^a \wedge e^+. \end{aligned} \tag{5}$$

The only non-vanishing component of the Ricci tensor is R_{++} given by

$$R_{++} = [(9-d)\alpha^2 + d\beta(2\alpha - \beta)] e^{-2\alpha x^+}, \tag{6}$$

and the Einstein equation $R_{++} = 0$ has two solutions

$$\beta = (1 \pm \frac{3}{\sqrt{d}})\alpha. \tag{7}$$

For $d = 1$, choosing the minus sign in (7) reproduces the background considered in [1].

We now show that the background given by eqs.(2) and (7) not only is a solution, but also preserves half of supersymmetries. The only interesting SUSY transformation is that for the gravitino $\delta\Psi_\mu = D_\mu \epsilon$. The components of spin connection of interest are ω_+, ω_i and ω_a:

$$\begin{aligned} \omega_+ &= -2\alpha - 2\alpha\gamma^-\gamma^+, \quad \omega_i = 2\alpha\gamma^i\gamma^+, \\ \omega_a &= 2\beta e^{(\beta-\alpha)x^+}\gamma^a\gamma^+. \end{aligned} \tag{8}$$

Since

$$[D_+, D_a] = \frac{1}{2}\beta(\beta - 2\alpha) e^{(\beta-\alpha)x^+}\gamma^a\gamma^+, \tag{9}$$

The compatibility condition for the constraints $D_\mu\epsilon = 0$ is $\gamma^+\epsilon = 0$. This condition eliminates half of the components in ϵ. The constraints $D_i\epsilon = D_a\epsilon = 0$ are solved if ϵ is independent of x^i and x^a. Finally, $D_+\epsilon = 0$ is solved if $\epsilon = \exp(\frac{1}{2}\alpha x^+)\eta$ for a constant η. We conclude that the unbroken supersymmetry is parametrized by η with the constraint $\gamma^+\eta = 0$. Thus, just like the original background of [1], our more general metric also preserves 16 supersymmtries, this should be enough to guarantee that our proposed matrix model to be described shortly is a valid description of dynamics over this background.

In the case $d = 9$, $\beta = 0, 2\alpha$, although $[D_+, D_a] = 0$, but $[D_+, D_i] = -\frac{1}{2}\alpha^2\gamma^i\gamma^+$, again we obtain the same unbroken supersymmetries.

In fact, (2) is only a special case of a more general class of solutions preserving half of supersymmetries (to be discussed in the end of this note). Before we study the matrix model for (2), let us discuss the geometric properties of (2). Apparently, the sign of α and its absolute value can be changed by changing x^+ and x^-, so we always assume $\alpha > 0$. As $x^+ \to -\infty$, the factor $e^{2\alpha x^+}$ approaches zero, the transverse dimensions x^i shrink to zero size, this is the big bang point with regard to these coordinates. This singularity locates a finite distance away in view of the affine parameter, indeed, let $dX^+ = e^{2\alpha x^+}dx^+$, the first term in the metric (2) becomes

$$-2dX^+dx^-, \tag{10}$$

thus X^+ is the affine parameter for the null geodesic x^- =const. The big bang singularity occurs at $X^+ = 0$.

The geometry of other transverse dimensions x^a depends on the choice β in (7). We name the choice $\beta = (1+3/\sqrt{d})\alpha$ case 1, and the choice $\beta = (1-3/\sqrt{d})\alpha$ case 2. For case 1, $\beta > 0$, so all x^a shrink at $x^+ = -\infty$. For case 2, $\beta < 0$ except for the $d = 9$ case, thus all x^a get to infinity at $x^+ = -\infty$ and shrink to zero size at $x^+ = \infty$. $d = 9$ is special, in this case, all 9 dimensions x^a do not evolve in time, and one can redefine x^+ such that the metric is Minkowski. Supersymmetry is also enhanced, since there is no longer constraint $\gamma^+\epsilon = 0$ following from (9).

The model studied in [1] is a special example of case 2 when $d = 1$. As in [1], we can compactify x^{10} on a circle to obtain type IIA string theory. The string metric and the dilaton ϕ are related to the M theory metric through

$$ds^2 = e^{-2\phi/3}ds^2_{st} + e^{4\phi/3}(dx^{10})^2, \tag{11}$$

we obtain

$$\phi = \frac{3}{2}\beta x^+, \quad ds^2_{st} = e^{(2\alpha+\beta)x^+}[-2dx^+dx^- + (dx^i)^2] + e^{3\beta x^+}(dx^a)^2. \tag{12}$$

The 10D Einstein metric reads

$$ds^2_E = e^{(2\alpha+\beta/4)x^+}[-2dx^+dx^- + (dx^i)^2] + e^{(9\beta/4)x^+}(dx^a)^2. \tag{13}$$

Since the string coupling "constant" $g_s = \exp(\phi) = \exp(\frac{3}{2}\beta x^+)$, for case 1, strings are weakly coupled at big bang, while all transverse dimensions are zero-sized, so it is not clear whether we can trust the perturbative string theory. As we shall see shortly in the matrix model, spacetime breaks down and we need to employ a full non-abelian description. At later times, strings become strongly coupled, while all dimensions expand, we shall see that a rather simple theory emerges, that is, only abelian degrees of freedom survive.

For case 2, strings are strongly coupled at big bang. transverse dimensions x^i start with a zero size, while transverse dimensions x^a have infinite size and contract as time evolves. This picture in valid both in terms of the string metric as well as the 10D Einstein metric. We will study in more detail the spacetime properties in the matrix model later.

The string frame metric (12) is in general non-flat. β never vanishes, so to get a flat metric, $d = 1$ is necessary, and in this case one chooses $\beta = -2\alpha$, this is the special case considered in [1]. More generally, one obtains a world-sheet theory with an action explicitly depending on time. One can attempt to quantize the string in the light-cone gauge, to do so, introduce the new light-cone coordinate $dy^+ = \exp((2\alpha + \beta)x^+)dx^+$, the first term in the string frame metric (12) becomes $-2dy^+dx^-$. The lignt-cone momentum $p_- = -p^+$ conjugate to x^- is conserved. In the light-cone quantization, we can set the light-cone gauge $y^+ = \tau$, the bosonic part of the world-sheet action reads

$$S = \frac{1}{4\pi\alpha'} \int d\tau d\sigma [e^{(2\alpha+\beta)x^+} \partial^\alpha X^i \partial_\alpha X^i + e^{3\beta x^+} \partial^\alpha X^a \partial_\alpha X^a], \tag{14}$$

where the period of σ is $2\pi\alpha' p^+$, and $\exp((2\alpha + \beta)x^+) = (2\alpha + \beta)\tau$. As $x^+ \to -\infty$, $\tau \to 0$. Since τ starts at a finite point, it is more useful to use the old light-cone coordinate $x^+ = t$, the action becomes

$$S = \frac{1}{4\pi\alpha'} \int dt d\sigma [e^{(4\alpha+2\beta)t} \partial^\alpha X^i \partial_\alpha X^i + e^{(2\alpha+4\beta)t} \partial^\alpha X^a \partial_\alpha X^a]. \tag{15}$$

The time-dependent coefficients may be interpreted as the effective tensions. For transverse coordinates X^i, the effective tension is $T_i = \frac{1}{2\pi\alpha'} e^{(4\alpha+2\beta)t}$, for coordinates X^a, the effective tension is $T_a = \frac{1}{2\pi\alpha'} e^{(2\alpha+4\beta)t}$. In case 1, both effective tensions get to zero at big bang, while the string coupling also gets to zero. Since the string spectrum becomes very dense, the usual free string picture does not apply. In later times, effective string tensions get large, and the string coupling constant also becomes strong, we will have to deal with a strongly coupled massless sector. In case 2, $4\alpha + 2\beta > 0$ except for $d = 1$, our above analysis still applies to T_i. $2\alpha + 4\beta = 6\alpha(1 - 2/\sqrt{d})$, for $d > 4$, T_a becomes small in

earlier time, the same as case 1. For $d < 4$, T_a becomes large in earlier time, thus these transeverse dimensions are effectively frozen. $d = 4$ is a special case, T_a is independent of time.

The vertex operator of a string state in the background (12) in general is quite complicated. For instance, consider a massless scalar satisfying equation of motion

$$\partial_\mu \left(e^{-2\phi} \sqrt{-g} g^{\mu\nu} \partial_\nu \Phi \right) = 0. \tag{16}$$

As function of x^+, the scaling of components g^{ab} is different from the scaling of components g^{+-} and g^{ij}, so there is no simple plane wave solution in general, which implies that the vertex operator of this massless scalar field is not simple. However, we can consider special cases when Φ is independent of x^a, in this case, a vertex operator $V = \exp(ik_+ x^+ + ik_- x^- + ik_i x^i)$ must satisfy the on-shell condition:

$$k_-(2k_+ - i\gamma) - k_i^2 = 0, \qquad \gamma = (9-d)\alpha + d\beta, \tag{17}$$

which require a imaginary part of k_+: $\Im k_- = \gamma/2$, thus in the vertex operator, there is an exponential factor

$$e^{-\frac{1}{2}\gamma x^+}. \tag{18}$$

Since $\gamma = (9 \pm 3\sqrt{d})\alpha \geq 0$, this exponential factor always blows up at $x^+ = -\infty$ for $d < 9$. The effective string coupling constant for these states is $g_{eff} = g_s \exp(-\frac{1}{2}\gamma x^+)$. Dimensions x^a for these states are effectively compactified, thus when we discuss interactions among these states, the space-time dimensionality is $11 - d$, and there is an effective string tension in action (15), or $\alpha'_{eff} = \alpha' \exp(-(4\alpha + 2\beta)x^+)$. The effective Newton constant is $G_{eff} = g_{eff}^2 \alpha'^{(9-d)/2}$, as a function of x^+, it scales

$$G_{eff} \sim e^{-\gamma x^+ - (9-d)(2\alpha+\beta)x^+}. \tag{19}$$

Now, γ is positive and $(9-d)(2\alpha+\beta)$ is non-negative if $d < 9$, the effective Newton constant blows up at $x^+ = -\infty$, the perturbative sstring picture is not valid at least for these states.

Next, we study the matrix model. It is not necessary to compactify any dimension of (2). In the usual flat background, the Matrix Theory action reads [7]

$$S = \int dt \mathrm{Tr} \left(\frac{1}{2R}(D_t X^i)^2 + \frac{R}{4}[X^i, X^j]^2 + i\theta^T D_t \theta - R\theta^T \gamma_i [X^i, \theta] \right), \tag{20}$$

where R is the longitudinal cut-off, or it may be viewed as the radius of x^- in the DLCQ M theory [8]. For simplicity, we set the M theory Planck length $l_p = 1$. A derivation of this matrix action is given in [9] (see also [10]). With our metric (2), it is straightforward to write the corresponding matrix action. To do this, we need to use the light-cone coordinate in (10), and identify X^+ with τ, the world-line time of D0-branes. In the matrix model, we apply the M theory metric (2) directly. Since the action is rather lengthy, we separate the action into the bosonic part and the fermionic part. The bosonic part reads

$$\begin{aligned} S_B = \int d\tau \mathrm{Tr}\{ \frac{1}{2R} e^{2\alpha x^+} (D_\tau X^i)^2 + \frac{1}{2R} e^{2\beta x^+} (D_\tau X^a)^2 + \frac{R}{4} e^{4\alpha x^+} [X^i, X^j]^4 \\ + \frac{R}{4} e^{4\beta x^+} [X^a, X^b]^4 + \frac{R}{2} e^{(2\alpha+2\beta)x^+} [X^i, X^a]^4 \}. \end{aligned} \tag{21}$$

Note that x^+ appearing in the above action is the old light-cone coordinate. The fermionic part reads

$$S_F = \int d\tau \{ i\theta^T D_\tau \theta - R e^{\alpha x^+} \theta^T \gamma_i [X^i, \theta] - R e^{\beta x^+} \theta^T \gamma_a [X^a, \theta] \}. \tag{22}$$

It is rather awkward to use τ as time, since it has a finite beginning. Let us switch back to the coordinate x^+ and on the world-line identify $t = x^+$, thus $d\tau = \exp(2\alpha t) dt$, we have

$$\begin{aligned} S_B = \int dt \mathrm{Tr}\{ \frac{1}{2R} (D_t X^i)^2 + \frac{1}{2R} e^{2(\beta-\alpha)t} (D_t X^a)^2 + \frac{R}{4} e^{6\alpha t} [X^i, X^j]^4 \\ + \frac{R}{4} e^{(2\alpha+4\beta)t} [X^a, X^b]^4 + \frac{R}{2} e^{(4\alpha+2\beta)t} [X^i, X^a]^4 \}, \end{aligned} \tag{23}$$

and

$$S_F = \int dt \{ i\theta^T D_t \theta - R e^{3\alpha t} \theta^T \gamma_i [X^i, \theta] - R e^{(2\alpha+\beta)t} \theta^T \gamma_a [X^a, \theta] \}. \tag{24}$$

Before study the simplest properties of this matrix model, let us show that we can recover the matrix string action of [1]. In this case, there is only one X^a, call it X^{10}. The compactification scheme is given in [11]. Up a dimensionful parameter, we replace the trace in the matrix action by $\int d\sigma \mathrm{Tr}$, the commutator $R[X^{10}, X^i]$ is replaced by the covariant derivative $iD_\sigma X^i$. Finally, use $\beta = -2\alpha$, we find

$$S_B = \int dt d\sigma \mathrm{Tr}\{ \frac{1}{2R} (D_\alpha X^i)^2 + \frac{1}{2R^3} g_s^2 F_{t\sigma}^2 + \frac{R}{4} g_s^{-2} [X^i, X^j]^2 \}, \tag{25}$$

and

$$S_F = \int dt d\sigma \mathrm{Tr}[\theta^T \sigma^\alpha D_\alpha \theta - R g_s^{-1} \theta^T \gamma_i [X^i, \theta], \tag{26}$$

where g_s is the time-dependent string coupling $g_s = \exp(-3\alpha t)$. Our action is identical to that in [1] up to an identification of a dimensionful parameter l_s. This matrix string theory can be regarded as a 2D Yang-Mills theory with a time-dependent coupling or a 2D Yang-Mills theory with a constant coupling in the world-sheet metric

$$ds^2 = g_s^{-2}(-dt^2 + d\sigma^2). \tag{27}$$

To study the properties of matrix model defined by (23) and (24), we consider cases 1 and 2 separately.

- Case 1, $\beta = (1 + \frac{3}{\sqrt{d}})\alpha$.

The kinetic term of X^i is always simple, as is the kinetic term of θ. The coefficients of the remaining terms all vanish in the limit $t \to -\infty$, so there is no constraint arising from these terms, this implies that all matrices are fully non-abelian. On the other hand, as $t \to \infty$, these coefficients blow up, thus, all matrices must commute with one another, the only surviving degrees of freedom are diagonal elements. Moreover, X^a must be independent of time in this limit, so X^a become frozen abelian moduli. Recall that in the string picture, we found previously that the effective tension T_a becomes infinitely heavy, even heavier than T_i for large t, this is related to the fact that X^a become moduli in the matrix model, while X^i still have dynamics. If we compactify some of the transverse dimensions, the story becomes slightly more involved. For instance, compactifying X^{10} on a circle, all other matrices become function on a circle σ. In the limit $t \to \infty$, although they have to be diagonal, they are not always periodic functions of σ, the eigen-values can get permuted after circling along σ, these twisted sectors describe strings of various lengths.

- Case 2, $\beta = (1 - \frac{3}{\sqrt{d}})\alpha$.

6α and $4\alpha + 2\beta$ are always positive, at big bang, there is no constraint on commutators $[X^i, X^j]$, $[X^i, X^a]$, $[X^i, \theta]$ and $[X^a, \theta]$. On the other hand, as $t \to \infty$, these commutators are forced to vanish. $2(\beta - \alpha)$ is always negative, so at bing bang, X^a must be independent of time, they become non-abelian moduli in the model. $2\alpha - 4\beta = 6\alpha(1 - 2/\sqrt{d})$, $d = 4$ becomes the critical dimension. For a larger d, there is no constraint on the commutators $[X^a, X^b]$ at big bang, and they have to vanish as $t \to \infty$, thus, for $d > 4$, all matrices commute in this limit and only abelian degrees of freedom survive. For $d < 4$, $[X^a, X^b]$ have to vanish at big bang, together with the fact that X^a are independent of time, these matrices become abelian moduli at big bang, this fact is also reflected in our previous analysis in the string picture, where we found T_a become infnitely heavy at big bang.

When $d=4$, X^a remain nonabelian all the time. In the string picture, recall that when $d=4$, the effective tension T_a is constant.

It is also of interest to study compactification. Just as in the flat background, we do not know how to write down matrix model action if we compactify more than 5 dimensions. We may choose to compactify all X^a, or some of X^a, or some of X^a and some of X^i. For simplicity, let us compactify all X^a on a torus T^d. Replacing $R[X^a, X^i]$ by iD_aX^i, $R^2[X^a, X^d]$ by $-F_{ab}$ etc, the bosonic part of the matrix model action reads

$$\begin{aligned} S_B = \int d^{d+1}\sigma\{ & \frac{1}{2R}(D_tX^i)^2 - \frac{1}{2R}e^{(4\alpha+2\beta)t}(D_aX^i)^2 + \frac{1}{2R^3}e^{2(\beta-2\alpha)t}F_{ta}^2 \\ & + \frac{1}{4R}e^{(2\alpha+4\beta)t}F_{ab}^2 + \frac{R}{4}e^{6\alpha t}[X^i, X^j]^2\}. \end{aligned} \tag{28}$$

The analysis of this action is similar to our previous analysis of the action without compactification, a statement there can be simply translated to one for the action (28). For instance, demanding $[X^a, X^b]=0$ is translated to $F_{ab}=0$, that is, the spatial connection must be flat.

It is interesting that the previous noticed "critical dimension" $d=4$ corresponds to the situation that the action (28) is no longer complete. Compactification on a torus T^4, the complete matrix model is to be given by the world volume theory of coincident M5-branes [12]. Of course, that theory will also be time-dependent, and hopefully one can figure out more details of the time dependence by studying field theory behavior of (28). Likewise, action (28) is again incomplete for compactification on T^5, in this case the complete theory is given by the little string theory [13].

Unlike the case when $d=1$, the Yang-Mills theory (28) can not be simply interpreted as a theory with a constant coupling on a world volume of a nontrivial metric. To interpret action (28) as Yang-Mills theory, we need to introduce both a nontrivial metric on the world volume as well as a time-dependent Yang-Mills coupling. Since a factor g_{YM}^{-2} appears in the coefficient of $F_{\mu\nu}^2$ and a factor g_{YM}^2 appears in the coefficient of $[X^i, X^j]^2$, we find for $d>1$

$$\begin{aligned} ds_{WV}^2 &= -e^{\frac{d}{d-1}(4\alpha+2\beta)t}dt^2 + e^{\frac{1}{d-1}(4\alpha+2\beta)t}(d\sigma^a)^2, \\ g_{YM}^2 &= e^{-\frac{1}{d-1}(4\alpha+2\beta)t+2(\alpha-\beta)t}, \end{aligned} \tag{29}$$

where the subscript WV stands for world volume. the world volume metric components g_{aa} must be all same, there are only 3 independent functions in the geometry and the Yang-Mills coupling, however, we need to match 5 different kinds of coefficients in (28), so it is nontrivial that there is a solution as presented in (29). This is quite important

especially for $d > 3$, since we need to complete the action (28) by introducing more degrees of freedom, this is possible only when (28) admits Yang-Mills theory interpretation. One can check that the identification (29) also works for the fermionic part of the matrix action.

Finally, we discuss generalizations of background (2). There are two directions to generalize (2). The first is to consider

$$ds^2 = -2e^{2\alpha x^+}dx^+dx^- + \sum_i e^{2\beta_i x^+}(dx^i)^2. \tag{30}$$

The Einstein equations are solved provided

$$\alpha^2 + \sum_i \beta_i(2\alpha - \beta_i) = 0. \tag{31}$$

This background again preserves 16 supersymmetries of the form $\epsilon = \exp(\frac{1}{2}\alpha x^+)\eta$, with a constant η satisfying $\gamma^+\eta = 0$.

The second direction to generalize (2) is to consider metric of the form

$$ds^2 = e^{2f(x^+)}(-2dx^+dx^- + (dx^i)^2) + e^{2g(x^+)}(dx^a)^2. \tag{32}$$

The non-vanishing curvature 2 forms are

$$\begin{aligned} R_{i+} &= (f'^2 - f'')e^{-2f}e^i \wedge e^+, \\ R_{a+} &= (2f'g' - g'^2 - g'')e^{-2f}e^a \wedge e^+. \end{aligned} \tag{33}$$

The Einstein equations are solved if

$$(9-d)(f'^2 - f'') + d(2f'g' - g'^2 - g'') = 0. \tag{34}$$

Once again, there are 16 unbroken supersymmetries, parameterized by $\epsilon = \exp(\frac{1}{2}f)\eta$, $\gamma^+\eta = 0$.

Needless to say that equation (34) admits infinite many solutions. Two special solutions deserve attention. One solution is when $g = 0$, in this case $f'^2 - f'' = 0$. All the curvature 2 forms vanish thus it may appear that we obtain a flat space solution. This is almost case except for a simple twist. The solution of $f'^2 - f'' = 0$ is $\exp(2f) = 1/(x^+)^2$, by changing coordinates $dy^+ = dx^+/(x^+)^2$, $y^- = x^-$ we obtain a metric

$$ds^2 = -2dy^+dy^- + (y^+)^2(dx^i)^2 + (dx^a)^2. \tag{35}$$

If one of x^i is periodic, we find a nontrivial background similar to the orbifold discussed in [2,3]. Of course if all x^i are noncompact, (35) is equivalent to the Minkowski space. Similarly, one may set $f = 0$ and find $\exp(2g) = (x^+)^2$, a nontrivial orbifold results provided one of x^a is compactified. One may attempt to construct a matrix model for the metric (35) too. However, time $\tau = y^+$ seems to have to terminate at $\tau = 0$. One may try to avoid this problem by going back to the original light-cone coordinate x^+ and take $t = x^+$ in the matrix model, although this helps to eliminate the coefficient τ^2 in the kinetic term of X^i but it also introduces a coefficient t^2 in the kinetic term of X^a, again t must be terminated at $t = 0$. Also the supersymmetry parameter $\epsilon = \sqrt{x^+}\eta$ does not exist beyond $x^+ = 0$ since ϵ has to be real. This problem may cause disease in the definition of the matrix model, and may be a reflection of the gravitational instability caused by a test particle.

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